Independence and Naturalness in Set-theoretic Practice (PhD Thesis)

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A problem about mathematical truth

- Common conception: Mathematical statements are either true or false.
- The set-theoretic independence phenomenon raises doubts on this conception.
- Set theory plays a foundational role for mathematics.
- Asset: its ability to handle infinite sets.
- But several questions are out of its range.
- Examples: the continuum hypothesis, Suslin's hypothesis, projective determinacy, the existence of large cardinals, etc.
- These statements are independent of the standard theory ZFC.

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The set-theoretic independence phenomenon

- Logical questions (e.g. about consistency) can be transformed into mathematical questions (Gödel coding).
- Consistency problems are thus solved by mathematicians, just like other mathematical problems.

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The set-theoretic knowledge of independent sentences (of ZFC) raises doubts about the conception that every mathematical statement is either true or false.

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Responses to the set-theoretic independence problem

- Many philosophers, mathematicians, and logicians have stated their diagnoses of this situation.
- Kurt Gödel (Gödel's programme):
  - ZFC should be extended by further axioms.
  - New axioms should be justified either intrinsically or extrinsically.
  - Intrinsic justification: fits with the iterative conception of set.
  - Extrinsic justification: appealing external factors, such as desirable consequences.

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The universe view and the multiverse view

- W. Hugh Woodin (Woodin's programme / the universe view):
  - There is a unique set-theoretic universe V.
  - In V, every sentence is either true or false.
  - Set theorists will find out which ones are true and which are false.
  - Woodin designs research programmes to achieve this goal.
- Joel D. Hamkins (the multiverse view):
  - Situation today: set-theoretic practitioners study set-theoretic models.
  - Most plausible explanation: There exists a multiverse encompassing all these set-theoretic models.
  - Set theorists explore this multiverse.

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Axiom justification in set-theoretic practice

Natural axioms

Significance of set-theoretic practice

The attitudes of set-theoretic practitioners towards axioms are decisive for solving the set-theoretic independence problem.

- Intrinsic and extrinsic justifications are abstract concepts; they are not part of the set-theoretic discourse.
- In the set-theoretic discourse: axioms are plausible, obvious, useful, or natural, etc.
- My dissertation: Focus on naturalness.
  - Set theorists make naturalness judgements on axioms.
  - A natural axiom is a plausible candidate for an acceptable axiom [Gödel 1947, Bagaria 2005].

Naturalness attempt

Find out which axioms set theorists find natural and propose them as additional axioms for ZFC.

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Research questions

Goal: Evaluate whether the naturalness attempt can work.

- The attitudes of set theorists towards the set-theoretic independence phenomenon
  - Philosophical views: Do set theorists have philosophical views about the independence phenomenon?
  - Disagreement: Do set theorists disagree on their philosophical views about the independence phenomenon? (such as Woodin and Hamkins)
- Requirements for the naturalness attempt
  - Acceptability: If set theorists find an axiom natural, do they also find it acceptable?
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Pragmatic approach

- Studying the decisive role of the attitudes of set theorists requires a pragmatic approach.
- Pragmatism: Philosophical claims are evaluated against empirical facts.
- Pragmatism applied to the set-theoretic independence problem:
  - If only a few set theorists believe in V (the multiverse), the universe view (multiverse view) is questioned.
  - If set theorists do not accept the axioms that they find natural, then the naturalness attempt does not work.
- Literature: conceptual analyses, ontological, and epistemological accounts; a pragmatic approach is missing.

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My project builds on ...

- the philosophy of mathematical practice: remove the tension between idealised philosophical concepts and their empirical counterparts (e.g. for notions like mathematical proof)
- social epistemology: knowledge production in groups (e.g. scientific communities), analyses of disagreement and agreement (e.g. peer disagreement and deep disagreement),
- the philosophy of set-theoretic practice
  - Penelope Maddy's work:
    - Philosophy must be with set-theoretic practice (Naturalism, Second Philosophy).
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- the philosophy of set-theoretic practice
  - Penelope Maddy's work:
    - Philosophy must fit with set-theoretic practice (Naturalism / Second Philosophy).
    - the universe view is correct.
    - Extrinsic justification of new axioms is valid.
  - Colin J. Rittberg's work
    - Maddy considers only a restricted part of set-theoretic practice; what about set theorists like Hamkins?
  - Hamkins' multiverse view: grounded in set-theoretic practice.

My method

- The accounts of Maddy, Rittberg, and Hamkins study a small number of specific perspectives.
- What is missing: An overview encompassing several different perspectives in the set-theoretic community.
- My method: Interview study with 28 set theorists from different research backgrounds (anonymised).
  - Interview questions addressed the research area, the use of new axioms, forcing, the possibility of extending ZFC by new axioms, naturalness judgements, and other issues.
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- 100 page-summary
- Collected data exceed presentation in a single publication.
- Analysed data in my dissertation refer to:
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Philosophical views of set theorists

Results

- Part of the sample (11 out of 28) endorses an absolutist view: believe in V and that ZFC will be extended by new axioms.
- Another part of the sample (11 out of 28) endorses a pluralist view: the concept of an intended model for set theory does not make sense, believe that no new axioms will be adopted, rather believe in ZFC as a conclusive theory for sets.
- Another part of the sample (6 out of 28) has more individual views that cannot be classified as absolutist or pluralist.
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Philosophical views

Answering the Philosophical views-question

- Philosophical views: Yes, the study suggests that set theorists often have determinate views on the set-theoretic independence problem, many of them either absolutist or pluralist, but also more individual views are possible.
  - Philosophical views and research areas:
    - forcing and cardinal characteristics may be predictive of a pluralist view,
    - descriptive set theory may be predictive of an absolutist or neither view;
    - other areas like inner model theory or forcing axioms were neutral, that is, not predictive of either view.

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Table: Philosophical views and research areas

Table: Philosophical views according to research areas

Total (interviewees)	[absolutist] 11	[pluralist] 11	Neither 6
Combinatorics	5	5	3
Descriptive set theory	5	1	5
Inner model theory	4	3	1
Forcing axioms	4	3	1
Large cardinals and forcing	3	4	1
Forcing	2	6	0
Set-theoretic/general topology	2	3	0
Cardinal characteristics	0	4	0

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Answering the Disagreement question

- Disagreement: Yes, the study suggests that set theorists with an absolutist and pluralist view disagree.
  - It is a deep disagreement:
    - about several interconnected propositions,
    - about epistemic principles: Absolutist practitioners consider desirable mathematical features of new axioms to be reasons in favour of these axioms, but pluralist practitioners do not consider desirable features to be reasons at all.
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Naturalness judgements

Results and answering the research questions

- Integral part of set-theoretic discourse.
- Often ambiguous, vague, or depend on non-semantic factors (e.g. time and research area).
- Positive connotation.
- Acceptability: No, the study suggests that only a few set theorists accept the axioms that they find natural (some of the absolutist practitioners, case study on forcing axioms).
- Agreement: No, the study suggests that there is no general agreement on naturalness judgements, but there can be in certain situations.
  - One can expect agreement only if set theorists make well-informed naturalness judgements.
  - Otherwise: differing naturalness judgements.
- Proposal: Naturalness judgements are a linguistic tool to assess the epistemic value of mathematical objects (rather than a tool to solve the set-theoretic independence problem).

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Value judgements

Generalisation from my findings on naturalness judgements

- Distinction between different discourse layers in set-theoretic discourse: one for mathematical propositions, one for value judgements, and one for philosophical beliefs.
- Thesis: although value judgements (desirability judgements) are neatly intertwined with philosophical claims when it comes to extrinsic justification, I argue that in general they can be separated.
- Arguments:
  - Participants of the study made this distinction ("I think the mathematical work of [Woodin/Hamkins] is valuable, but I disagree with their philosophical views").
  - Absolutist and pluralist participants agree on many value judgements, while disagreeing on their philosophical views (results on surprising theorems).
- Value judgements play a significant role for set-theoretic progress, disregarding the philosophical views of involved set theorists.

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Conclusion

Evaluation of the naturalness attempt

- Goal: Evaluate whether the naturalness attempt can work.
- Pragmatic insights into the set-theoretic independence problem.
- Set theorists—who are experts on set-theoretic independence—have determinate views on the independence problem.
- The views can differ largely: absolutist and pluralist views.
- From a pragmatic perspective: the naturalness attempt does not work.
- Reason: There are too many set theorists (those with pluralist views) who will not adopt further axioms (whether or not they find them natural).
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Pragmatic conclusion on the set-theoretic independence problem

Will the set-theoretic community adopt axioms beyond ZFC?

- Despite the disagreements, my book also shows that the set-theoretic discourse involving naturalness judgements is characterised by discussion, comprehension, and approximation of judgements, and significant for set-theoretic progress.
- Projections into the future based on this current snapshot of set-theoretic practice:
  - Either the situation remains as it is now: the community as a whole will not adopt further axioms, because there are set theorists (with pluralist views) who will not accept any axioms beyond ZFC.
  - Or: substantial advances in set-theoretic knowledge and understanding will have the power to fundamentally change the situation (comparable to the introduction of forcing). Looking over the achievements of the last century, it is not unlikely that similar things could happen in the next century.

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Pragmatic conclusion on the set-theoretic independence problem

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Sample set

Research areas of the interviewees

The interviewees indicated between one and five research areas:

Table: Distribution of main research areas

Combinatorics	13
Descriptive set theory	11
Ergodic theory	4
Inner model theory	8
Forcing axioms	8
Large cardinals and forcing	8
Forcing	8
Set-theoretic and general topology	5
Cardinal characteristics	4

Sample set

Further characteristics

- Interviewees have/had a permanent position as a professor of mathematics with research focus on set theory.
- Year of obtaining the PhD:

before 1980	1980–1989	1990–1999	after 1999
6	4	9	9

- 24 from 28 are men.
- Most affiliations in Europe (15) or USA (11)
- Conclusion: The sample is diverse and not biased in obvious ways.

Milestone theorems are valued by absolutist and pluralist practitioners

Interview question: "Do you remember any surprising results in the history of set theory, that was either surprising for you or for the community?"

Table: Surprising theorems with more than two indications

	[pluralist]	[absolutist]	Neither
Total (interviewees)	11	11	6
Determinacy principles and LCAs	2	5	0
The introduction of forcing	1	4	1
Shelah's pcf theory	2	3	0
$\mathfrak{p} = \mathfrak{t}$	2	1	1