CLPS CENTRE FOR LOGIC & PHILOSOPHY OF SCIENCE RESEARCH GROUP

fwo For funding of some offsprings of the PhD project, in my postdoc (FWOAL950).

FORMAL REPRESENTATION AND MATHEMATICAL PRACTICE:

CAPTURING DYNAMICS OF AXIOMATIZATIONS WITH METHODS FROM PHILOSOPHY OF SCIENCE

Deniz Sarikaya
Universität zu Lübeck &
Vrije Universiteit Brussel

CL Vienna: October 7, 2024

AGENDA

Biographical: 1 min

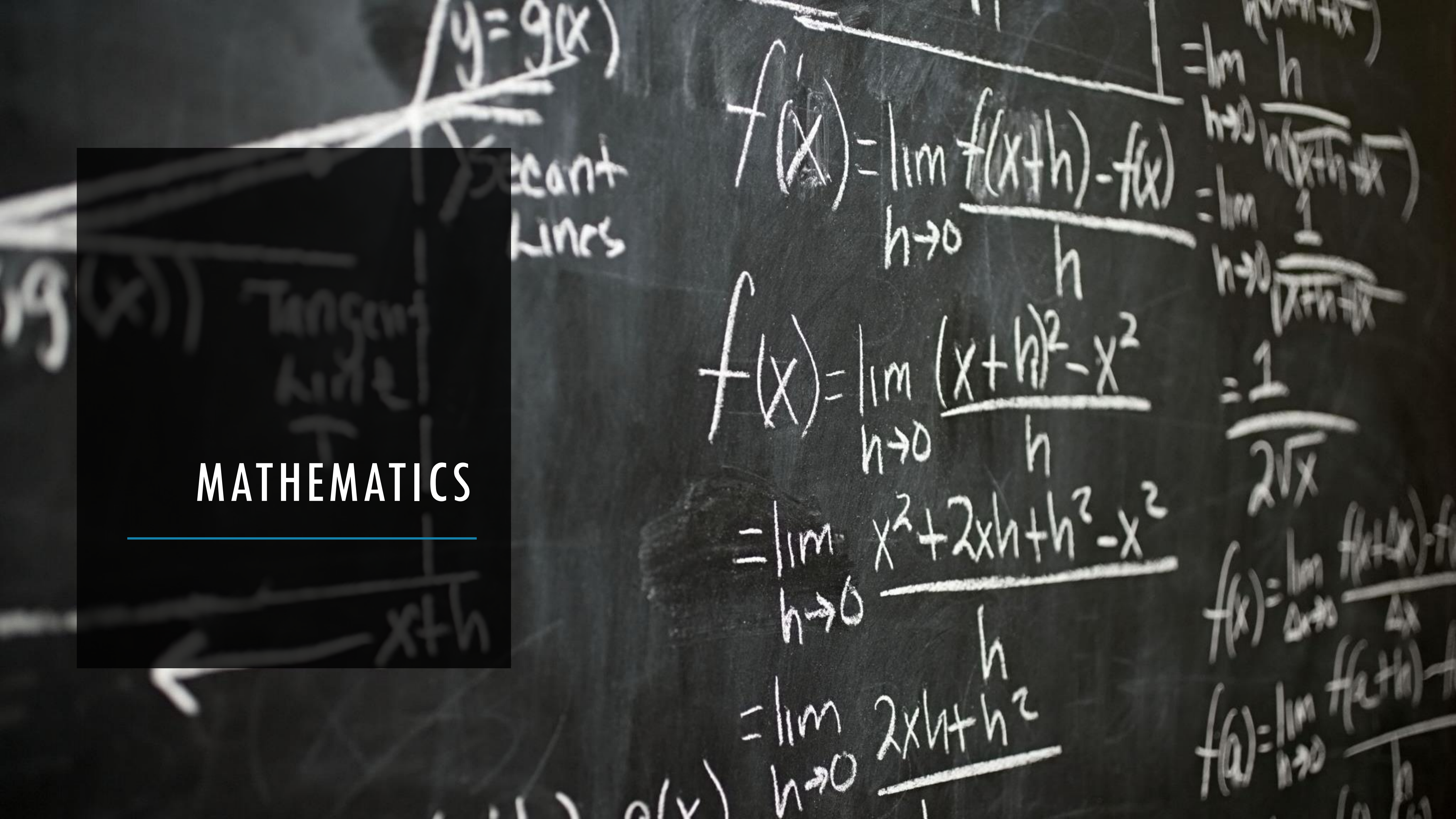
Overview: 5 min

Frames and paradigmatic examples: 7 + 7

FROM HAMBURG

BRUSSELS

LÜBECK

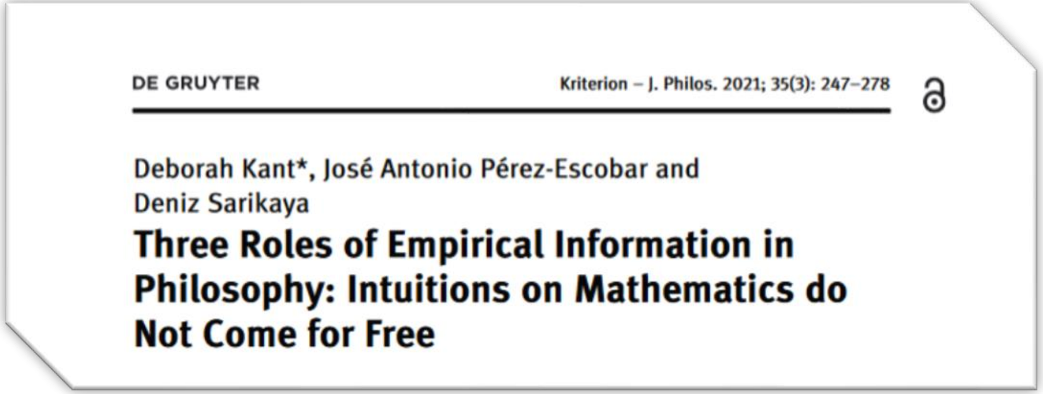


MATHEMATICS

PHILOSOPHY

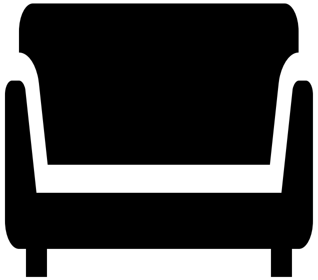
OVERVIEW

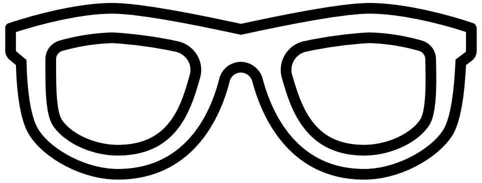
CHAPTER 2:


DE GRUYTER Kriterion – J. Philos. 2021; 35(3): 247–278

Deborah Kant*, José Antonio Pérez-Escobar and
Deniz Sarikaya

Three Roles of Empirical Information in Philosophy: Intuitions on Mathematics do Not Come for Free





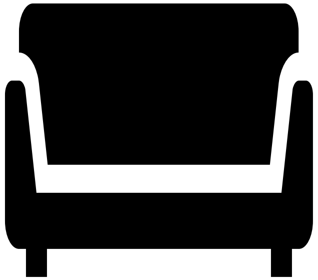
• • •

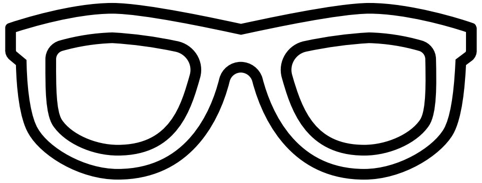
CHAPTER 2:

DE GRUYTER Kriterion – J. Philos. 2021; 35(3): 247–278

Deborah Kant*, José Antonio Pérez-Escobar and Deniz Sarikaya

Three Roles of Empirical Information in Philosophy: Intuitions on Mathematics do Not Come for Free





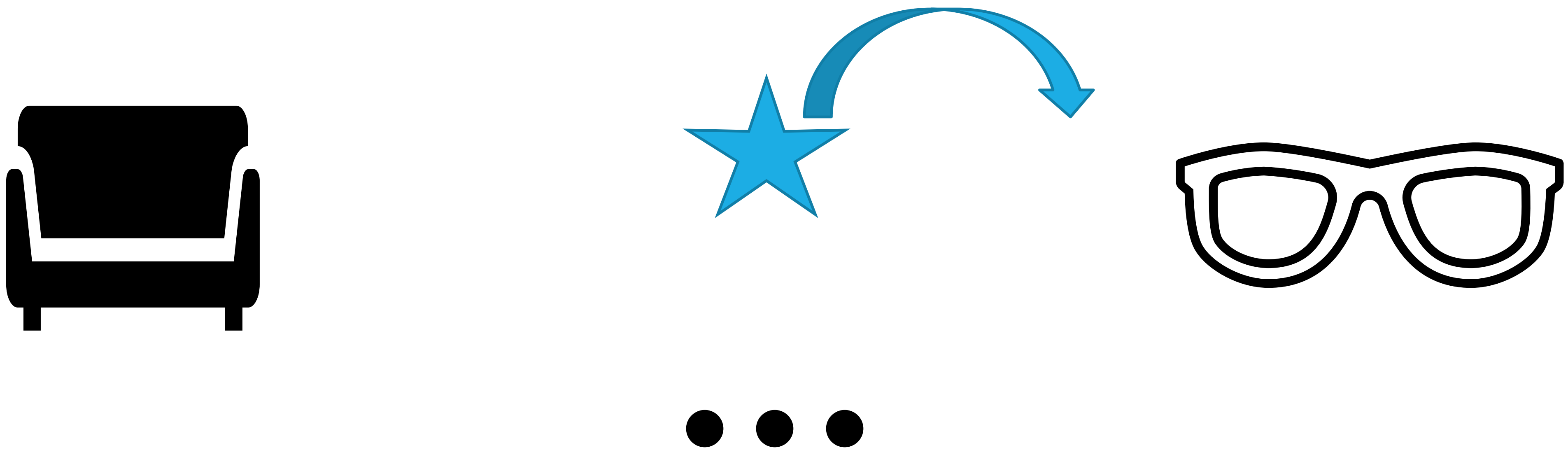
...

CHAPTER 2:

DE GRUYTER Kriterion – J. Philos. 2021; 35(3): 247–278

Deborah Kant*, José Antonio Pérez-Escobar and
Deniz Sarikaya

Three Roles of Empirical Information in Philosophy: Intuitions on Mathematics do Not Come for Free



CHAPTER 3:

Synthese (2021) 199:3405–3429
https://doi.org/10.1007/s11229-020-02939-y

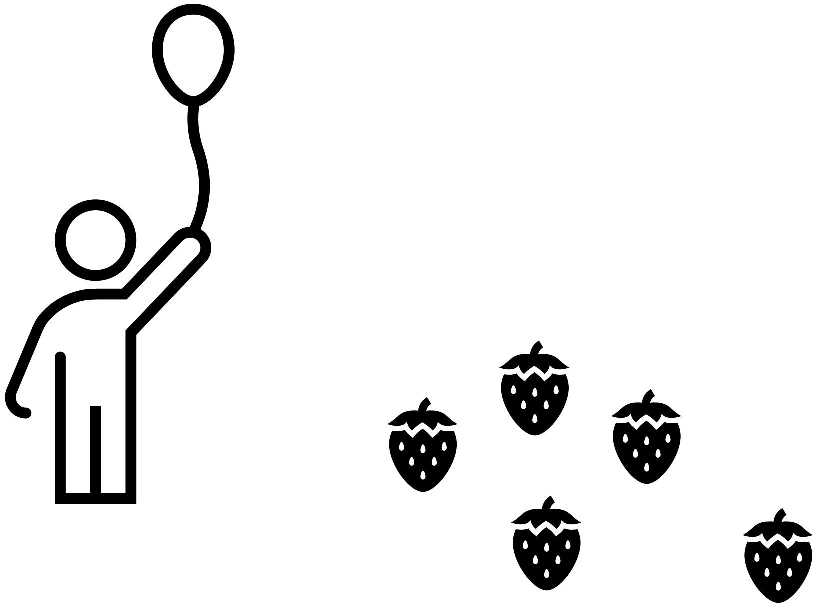
VIRTUE THEORY OF MATHEMATICAL PRACTICES

Check for updates

Mathematizing as a virtuous practice: different narratives and their consequences for mathematics education and society

Deborah Kant[1] · Deniz Sarikaya[2]

ived: 1 October 2019 / Accepted: 26 October 2020 / Published online: 7 November 2020



CHAPTER 3:

Synthese (2021) 199:3405–3429
https://doi.org/10.1007/s11229-020-02939-y

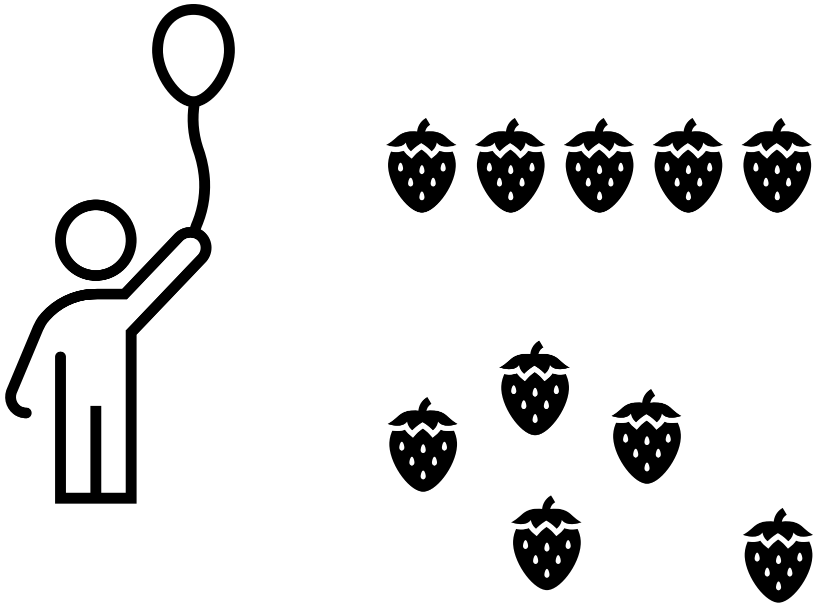
VIRTUE THEORY OF MATHEMATICAL PRACTICES

Check for updates

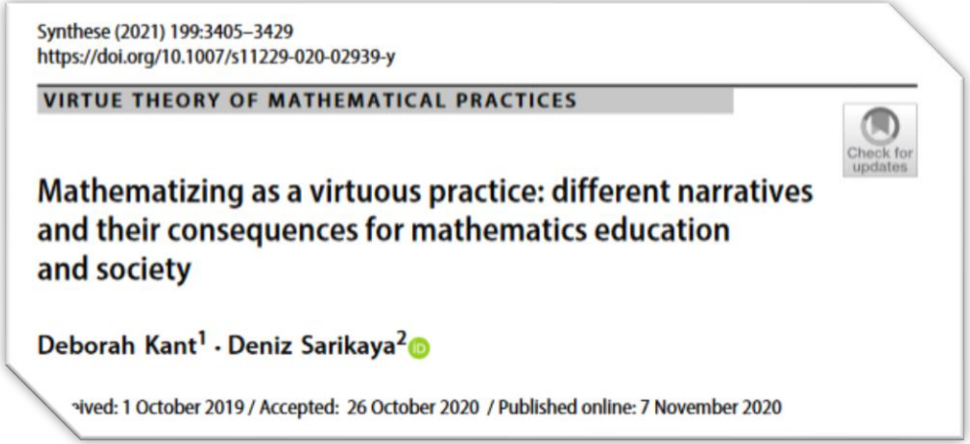
Mathematizing as a virtuous practice: different narratives and their consequences for mathematics education and society

Deborah Kant[1] · Deniz Sarikaya[2]

ived: 1 October 2019 / Accepted: 26 October 2020 / Published online: 7 November 2020



CHAPTER 3:



Synthese (2021) 199:3405–3429
https://doi.org/10.1007/s11229-020-02939-y

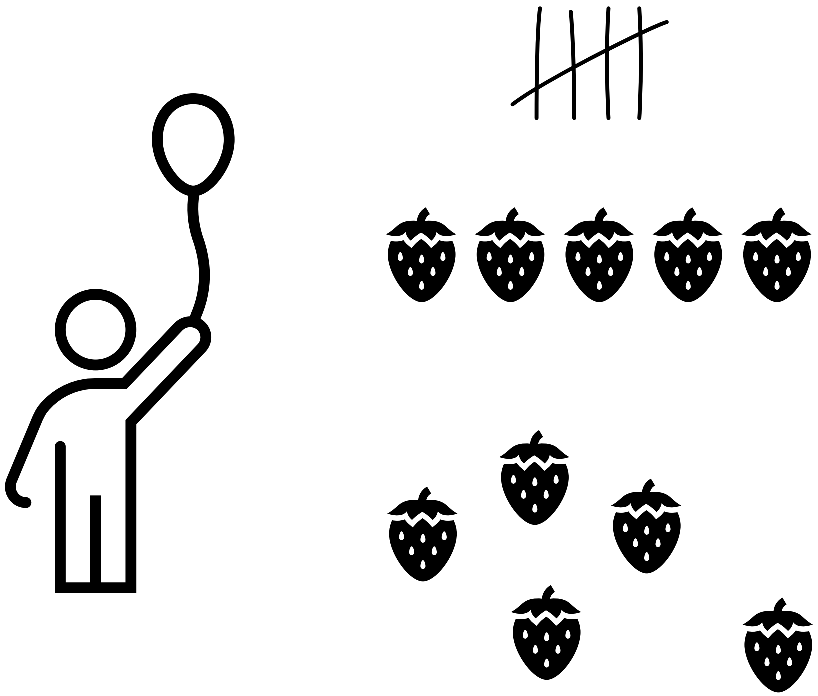
VIRTUE THEORY OF MATHEMATICAL PRACTICES

Check for updates

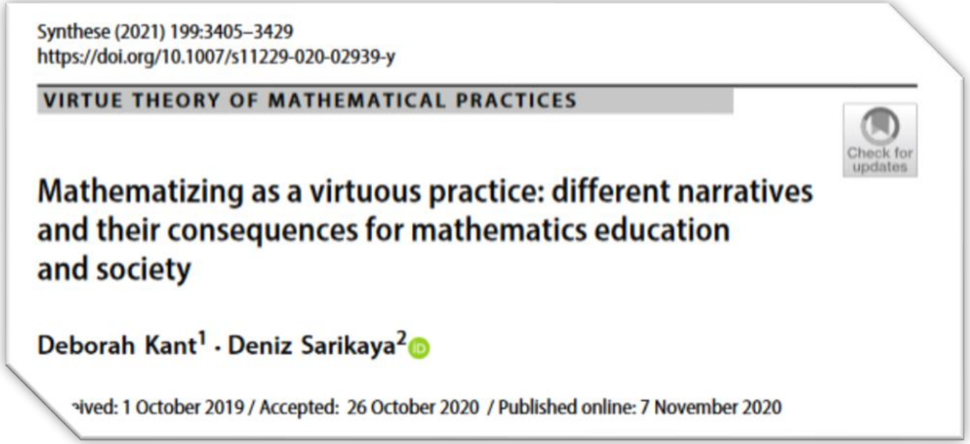
Mathematizing as a virtuous practice: different narratives and their consequences for mathematics education and society

Deborah Kant[1] · Deniz Sarikaya[2]

ived: 1 October 2019 / Accepted: 26 October 2020 / Published online: 7 November 2020



CHAPTER 3:


Synthese (2021) 199:3405–3429
https://doi.org/10.1007/s11229-020-02939-y

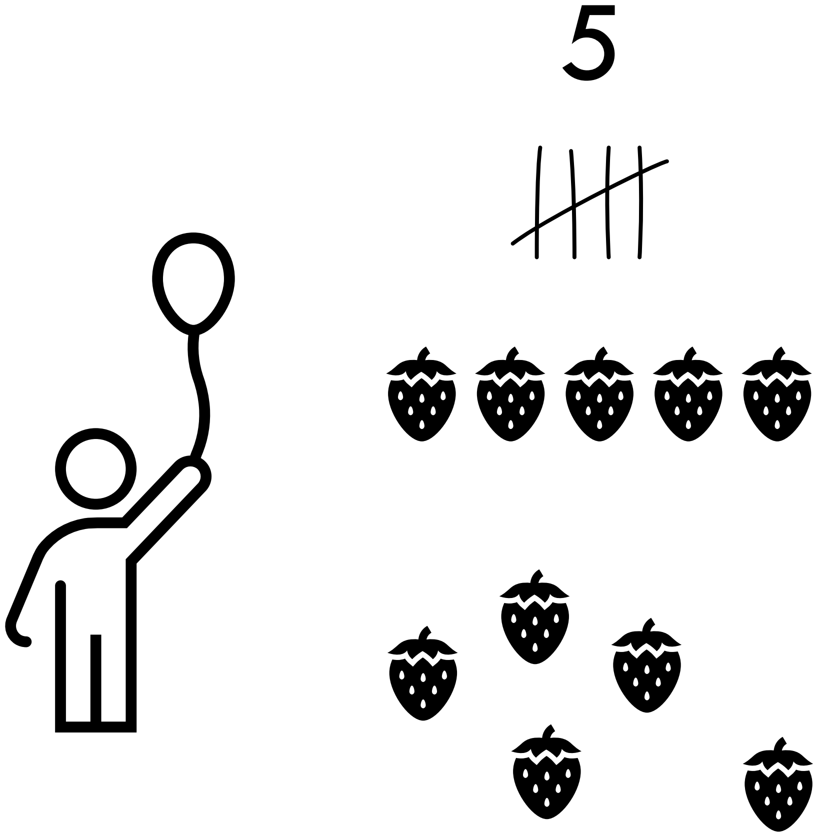
VIRTUE THEORY OF MATHEMATICAL PRACTICES

Check for updates

Mathematizing as a virtuous practice: different narratives and their consequences for mathematics education and society

Deborah Kant[1] · Deniz Sarikaya[2]

·ived: 1 October 2019 / Accepted: 26 October 2020 / Published online: 7 November 2020



CHAPTER 4


European Journal for Philosophy of Science (2022) 12: 1
https://doi.org/10.1007/s13194-021-00435-9

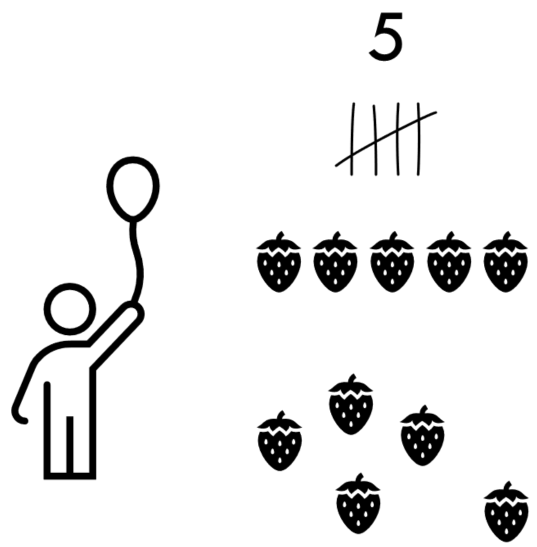
PAPER IN PHILOSOPHY OF SCIENCE IN PRACTICE

Check for updates

Purifying applied mathematics and applying pure mathematics: how a late Wittgensteinian perspective sheds light onto the dichotomy

José Antonio Pérez-Escobar[1] · Deniz Sarikaya[2]

ived: 16 June 2020 / Accepted: 6 December 2021 / Published online: 23 December 2021



Embodied world

CHAPTER 4


European Journal for Philosophy of Science (2022) 12: 1
https://doi.org/10.1007/s13194-021-00435-9

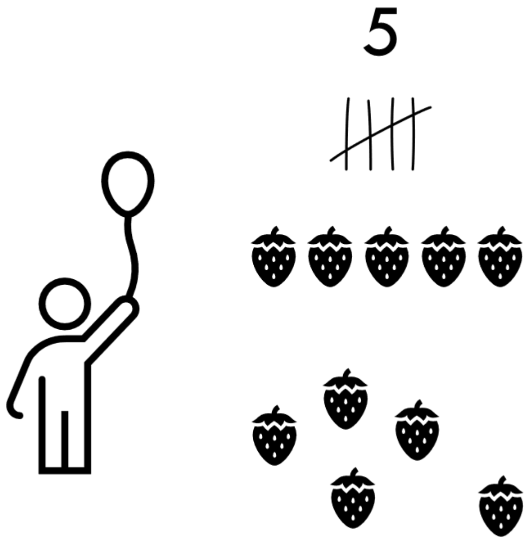
PAPER IN PHILOSOPHY OF SCIENCE IN PRACTICE

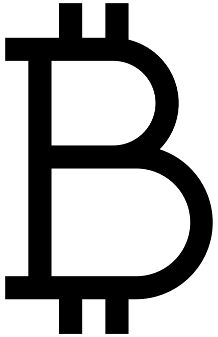
Check for updates

Purifying applied mathematics and applying pure mathematics: how a late Wittgensteinian perspective sheds light onto the dichotomy

José Antonio Pérez-Escobar[1] · Deniz Sarikaya[2]

ived: 16 June 2020 / Accepted: 6 December 2021 / Published online: 23 December 2021





Embodied world

theory building/ (unintended) application

CHAPTER 4

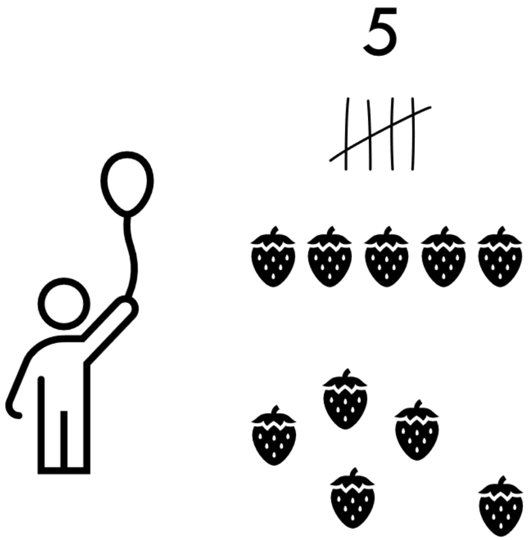
European Journal for Philosophy of Science (2022) 12: 1
https://doi.org/10.1007/s13194-021-00435-9

PAPER IN PHILOSOPHY OF SCIENCE IN PRACTICE

Purifying applied mathematics and applying pure mathematics: how a late Wittgensteinian perspective sheds light onto the dichotomy

José Antonio Pérez-Escobar[1] · Deniz Sarikaya[2]

ived: 16 June 2020 / Accepted: 6 December 2021 / Published online: 23 December 2021

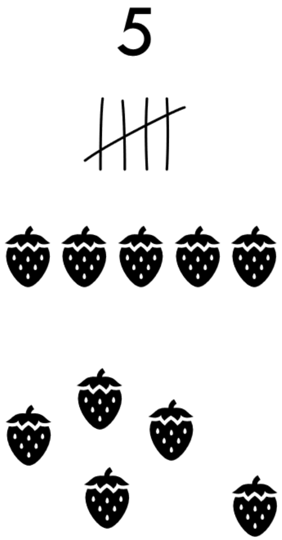


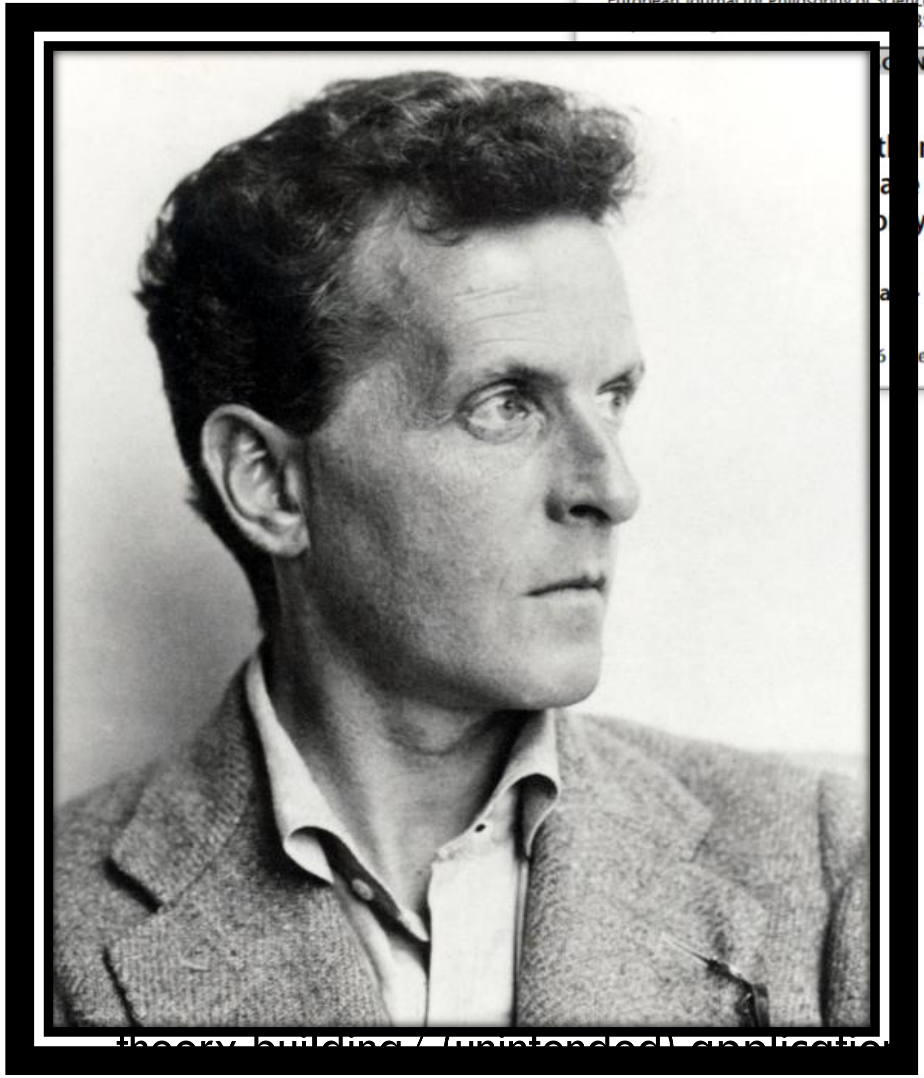
Embodied world

theory building/ (unintended) application

import

CHAPTER 4





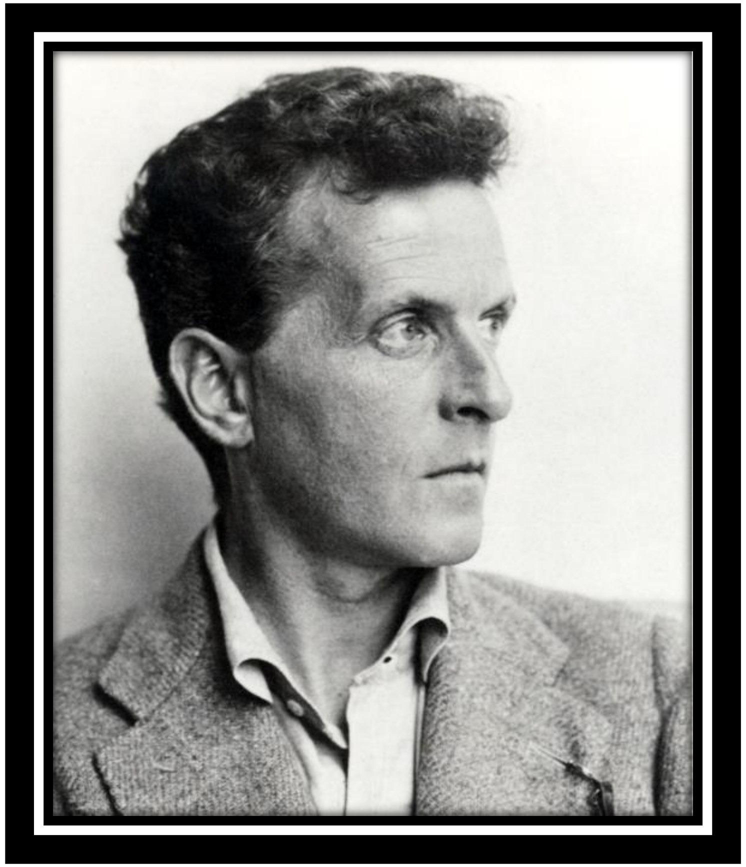
Embodied world

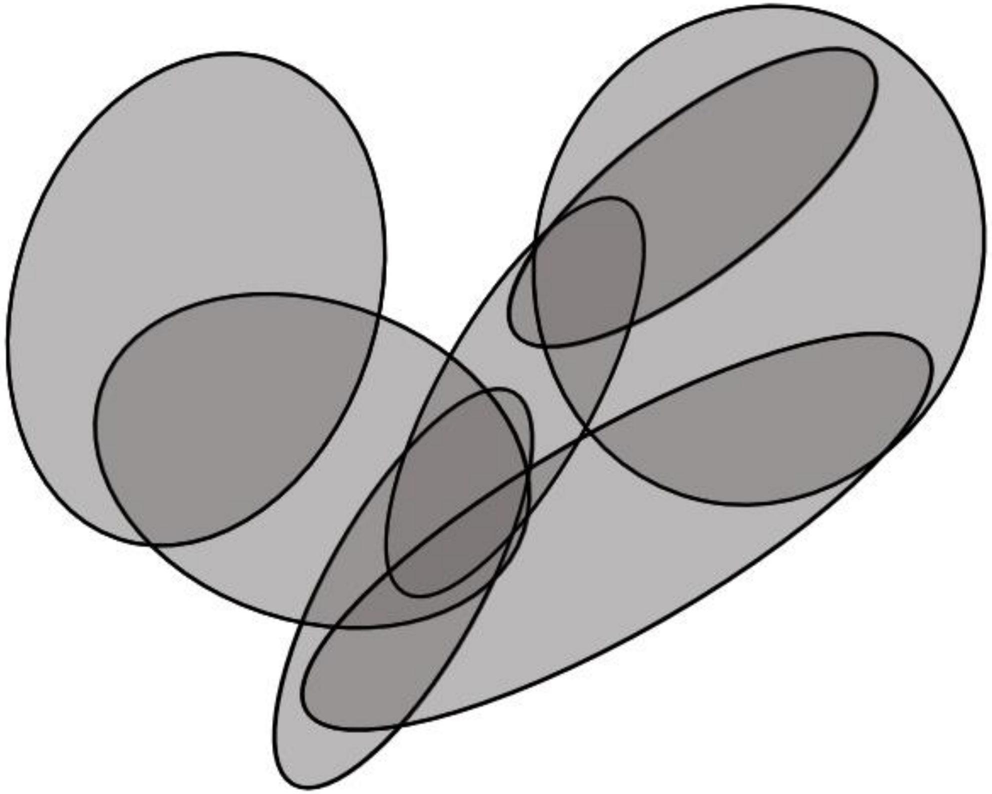
theory building / (unintended) applicatio

import

Ludwig Wittgenstein (1899–1951). Österreichische Nationalbibliothek, Inventarnummer Pf 42.805 : C (1)

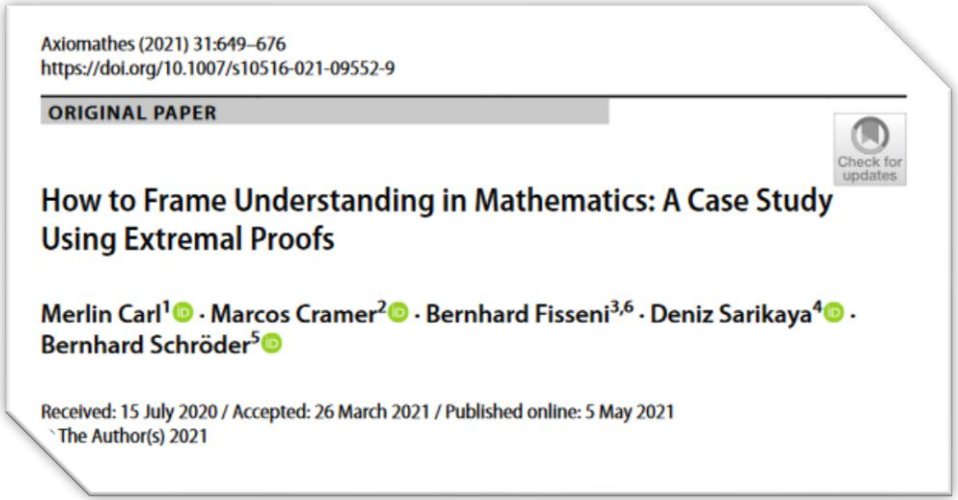
CHAPTER 5 (TO BE RESUBMITTED)





Ludwig Wittgenstein (1899–1951). Österreichische Nationalbibliothek, Inventarnummer Pf 42.805 : C (1)

CHAPTER 6



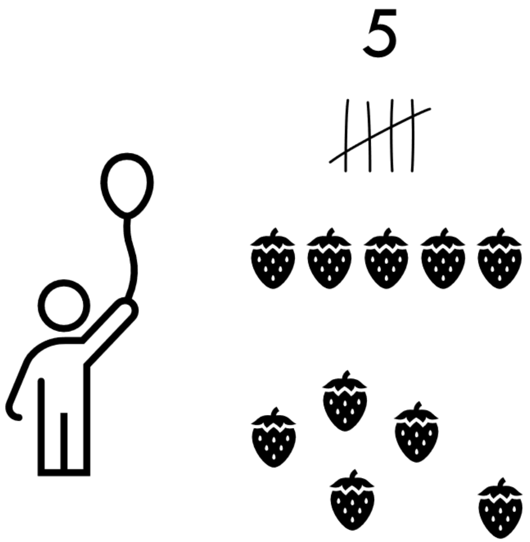
Axiomathes (2021) 31:649–676
https://doi.org/10.1007/s10516-021-09552-9

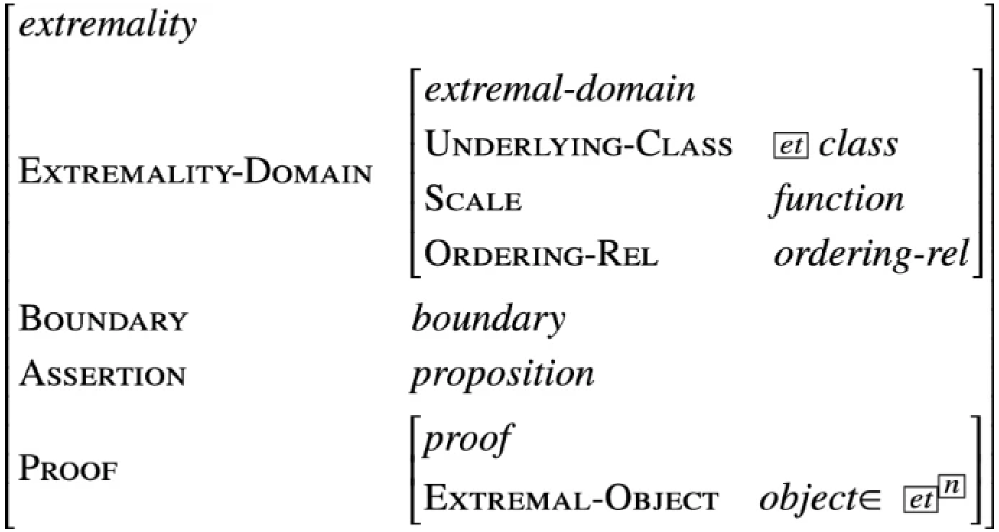
ORIGINAL PAPER

How to Frame Understanding in Mathematics: A Case Study Using Extremal Proofs

Merlin Carl[1] · Marcos Cramer[2] · Bernhard Fisseni[3,6] · Deniz Sarikaya[4] · Bernhard Schröder[5]

Received: 15 July 2020 / Accepted: 26 March 2021 / Published online: 5 May 2021
© The Author(s) 2021





CHAPTER 7


Synthese (2023) 202:108
https://doi.org/10.1007/s11229-023-04310-3

ORIGINAL RESEARCH

Check for updates

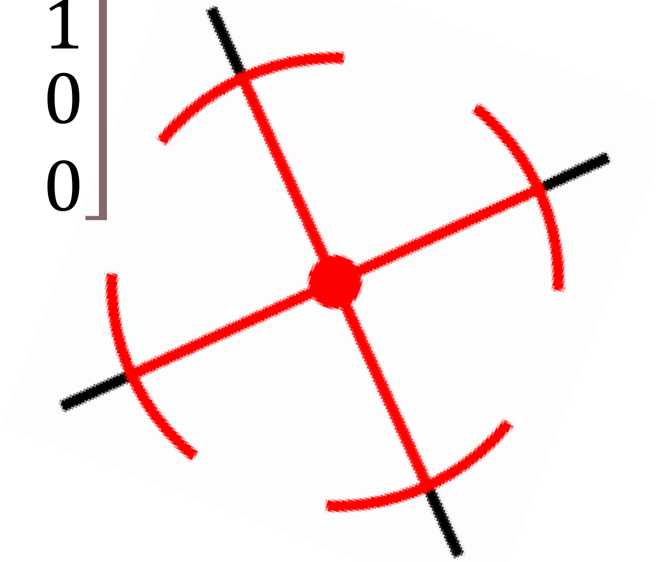
How to frame innovation in mathematics

Bernhard Fisseni[1] · Deniz Sarikaya[2] · Bernhard Schröder[1]

Received: 17 June 2022 / Accepted: 20 July 2023 / Published online: 25 September 2023
© The Author(s) 2023

extremality		
EXTREMALITY-DOMAIN	*extremal-domain*	
	UNDERLYING-CLASS	[et] *class*
	SCALE	*function*
	ORDERING-REL	*ordering-rel*
BOUNDARY	*boundary*	
ASSERTION	*proposition*	
PROOF	*proof*	
	EXTREMAL-OBJECT	*object* ∈ $[et]^n$

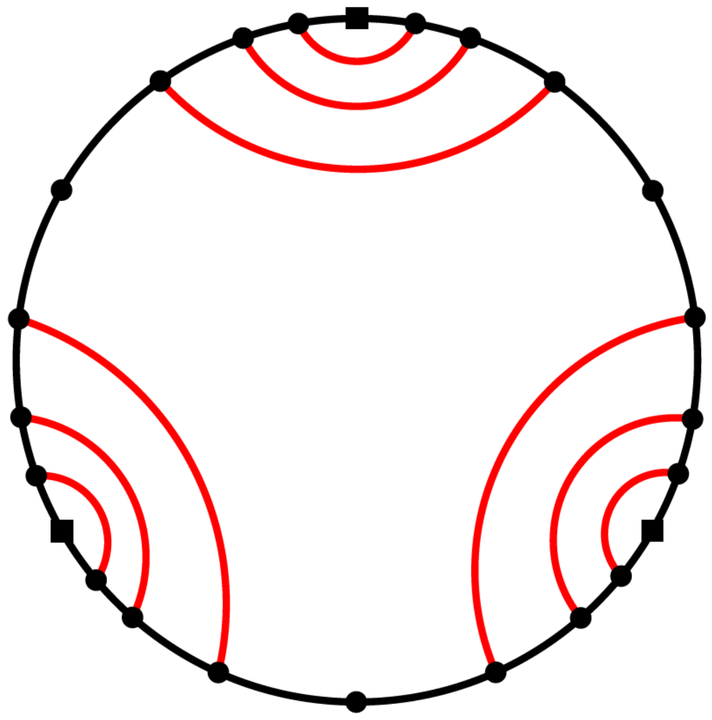
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$



CHAPTER 8

The making of a mathematical object

extremality		
EXTREMALITY-DOMAIN	*extremal-domain*	
	UNDERLYING-CLASS	[et] *class*
	SCALE	*function*
	ORDERING-REL	*ordering-rel*
BOUNDARY	*boundary*	
ASSERTION	*proposition*	
PROOF	*proof*	
	EXTREMAL-OBJECT	*object*$\in$ $[et]^n$



Frames

and paradigmatic examples

What are frames

Properties

- a concept in knowledge representation
- represent conceptual structure or prototypical situations e.g. ***birthday celebration***, ***restaurant***.
- ***roles*** and ***participants*** (slots and fillers) e.g. *waiter*, *diners*, *food*, …
- organized in an ***inheritance hierarchy typed*** *feature structures*

Usage

- e.g., in cognitive linguistics and artificial intelligence
- explain how receiver completes information conveyed by sender
- linguistic project: FrameNet database (1,200 semantic frames)
- Originates from MIT

Frames and Framing

Frame: BUYING

[Buyer John] **bought** [Goods a beautiful medieval book] [Time yesterday] .

Frame: SELLING

[Seller Petra] **sold** [Goods a beautiful medieval book] to [Buyer John] for [Money twenty Euros] .

Frames and feature structures

(2) a.

$$\begin{bmatrix} \textit{buy} & \\ \textsc{Buyer!} & [\![\textit{John}]\!] \\ \textsc{Goods!} & [\![\textit{a beautiful medieval book}]\!] \\ \textsc{Time} & [\![\textit{yesterday}]\!] \\ \textsc{Seller} & \textit{person} \\ \textsc{Money} & \textit{money} \\ \textsc{Purpose} & \textit{purpose} \\ \ldots & \end{bmatrix} = \begin{bmatrix} \textit{buy} & \\ \textsc{Buyer!} & j \\ \textsc{Goods!} & b \\ \textsc{Time} & \begin{bmatrix} \textit{point-in-time} & \\ \textsc{year} & \textit{2018} \\ \textsc{month} & \textit{02} \\ \textsc{day} & \textit{28} \\ \textsc{hour} & \{1, \ldots, 24\} \\ \textsc{minute} & \{0, \ldots, 60\} \\ \ldots & \end{bmatrix} \\ \textsc{Seller} & \textit{person} \\ \textsc{Money} & \textit{money} \\ \textsc{Purpose} & \textit{purpose} \\ \ldots & \end{bmatrix}$$

b.

$$\begin{bmatrix} \textit{sell} & \\ \textsc{Seller!} & [\![\textit{Peter}]\!] \\ \textsc{Buyer!} & [\![\textit{John}]\!] \\ \textsc{Goods!} & [\![\textit{a beautiful medieval book}]\!] \\ \textsc{Time} & \textit{point-in-time} \\ \textsc{Money} & [\![\textit{twenty Euros}]\!] \\ \textsc{Purpose} & \textit{purpose} \\ \ldots & \end{bmatrix} = \begin{bmatrix} \textit{sell} & \\ \textsc{Seller!} & p \\ \textsc{Buyer!} & j \\ \textsc{Goods!} & b \\ \textsc{Time} & \textit{point-in-time} \\ \textsc{Money} & \textit{20€} \\ \textsc{Purpose} & \textit{purpose} \\ \ldots & \end{bmatrix}‘$$

A look into the framenet

Frame Index

A B C D E F G H I J K L M N O P Q R S
T U V W X Y Z

Abandonment
Abounding_with
Absorb_heat
Abundance
Abusing
Access_scenario
Accompaniment
Accomplishment
Accoutrements
Accuracy
Achieving_first
Active_substance
Activity
Activity_abandoned_state
Activity_done_state
Activity_finish
Activity_ongoing
Activity_pause
Activity_paused_state
Activity_prepare
Activity_ready_state
Activity_resume
Activity_start
Activity_stop
Actually_occurring_entity
Addiction
Adding_up
Adducing
Adjacency
Adjusting
Adopt_selection
Aesthetics

Commercial_transaction

Lexical Unit Index

Definition:

These are words that describe basic commercial transactions involving a Buyer and a Seller who exchange Money and Goods. The individual words vary in the frame element realization patterns. For example, the typical patterns for the verbs buy and sell are: BUYER buys GOODS from the SELLER for MONEY. SELLER sells GOODS to the BUYER for MONEY.

His $20 TRANSACTION with Amazon.com for a new TV had been very smooth.

FEs:

Core:

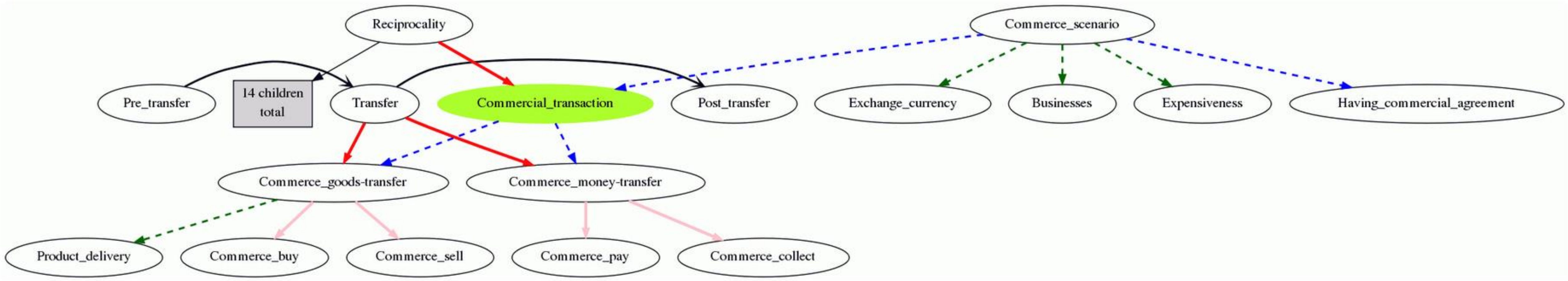
Buyer [Byr]	The Buyer wants the Goods and offers Money to a Seller in exchange for them.
Goods [Gds]	The FE Goods is anything (including labor or time, for example) which is exchanged for Money in a transaction.
Money [Mny]	Money is the thing given in exchange for Goods in a transaction.
Seller [Slr]	The Seller has possession of the Goods and exchanges them for Money from a Buyer.

Non-Core:

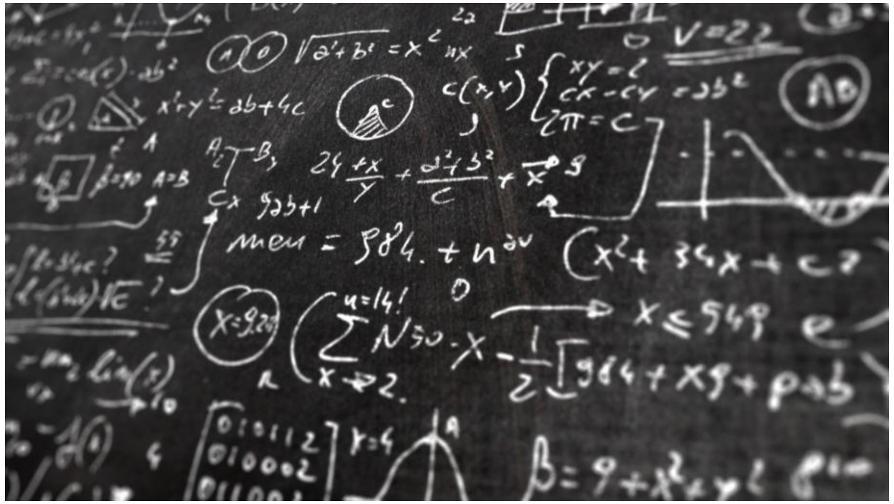
Means [Mns] **Semantic Type:** State_of_affairs	The means by which a commercial transaction occurs.
Rate [Rate]	Price or payment per unit of Goods.
Unit [Unit]	The Unit of measure of the Goods according to which the exchange value of the Goods (or services) is set. Generally, it occurs in a by-PP.

Frame-frame Relations:

A look into the framenet



- Screenshot https://framenet.icsi.berkeley.edu/fndrupal/FrameGrapher



Frames for mathematical texts

Frames in Mathematical Texts

- **Goal:** Model proofs and proof methods

- **Types of frames:** (define types of slots)

  Ontological: type of mathematical object

  e.g. **Circle**, **slots:** center, radius, diameter, circumference, …

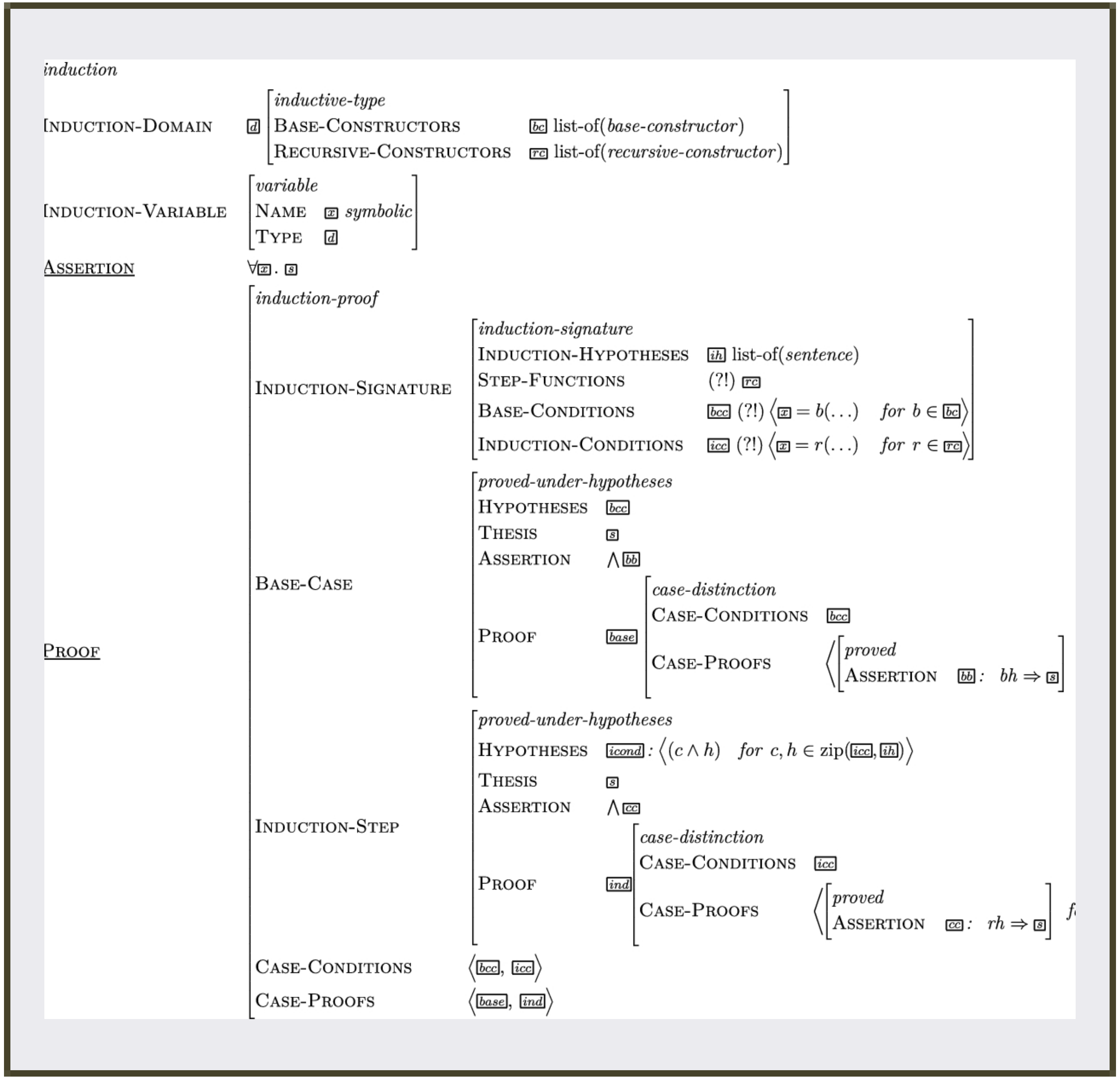
  e.g. **Vector Space**, **slots:** zero, unit, field, dimension, …

- **Structural:** part of proofs

  e.g. **Induction**, **slots:** induction variable, hypothesis, step, domain, . . .

  e.g. **Extremal Proof**, **slots:** object type, initial object, parameter

The Induction Frame



Proof. First, the second statement is indeed more precise than the first: let $k \geq 1$ be such that $f^k = 0$ but $f^{k-1} \neq 0$; there exists $v \neq 0$ such that $f^{k-1}(v) \neq 0$, and we obtain $k \leq n$ by applying the second result to this vector v. We now prove the second claim. Assume therefore that $v \neq 0$ and that $f^k(v) = 0$ but $f^{k-1}(v) \neq 0$. Let $t_0, \ldots, t_{k-1}$ be elements of $\mathbf{K}$ such that

$$t_1 v + \cdots + t_{k-1} f^{k-1[sic!]}(v) = 0.$$

Apply f^{k-1} to this relation; since $f^k(v) = \ldots = f^{2k-2[sic!]}(v) = 0$, we get

$$t_1 f^{k-1}(v) = t_1 f^{k-1}(v) + t_2 f^k(v) + \cdots + t_{k-1} f^{2k-2[sic!]}(v) = 0,$$

and therefore $t_1 f^{k-1}(v) = 0$. Since $f^{k-1}(v)$ was assumed to be non-zero, it follows that $t_1 = 0$. Now repeating this argument, but applying f^{k-2} to the linear relation (and using the fact that $t_1 = 0$), we get $t_2 = 0$.
Then similarly we derive by induction that $t_i = 0$ for all i, proving the linear independence stated.

induction

- INDUCTION-DOMAIN: [d] *inductive-type*
  - BASE-CONSTRUCTOR: [bc] *base-constructor*
  - RECURSIVE-CONSTRUCTOR: [rc] *recursive-constructor*
- INDUCTION-VARIABLE: *variable*
  - NAME: [x] *symbolic*
  - TYPE: [d]
- ASSERTION: ∀[x] . [s]
- PROOF: *induction-proof*
  - INDUCTION-SIGNATURE: *induction-signature*
    - INDUCTION-HYPOTHESIS: [ih] *sentence*
    - STEP-FUNCTION: (?!) [rc]
    - BASE-CONDITION: [bcc] (?!) [x] = [bc]
    - INDUCTION-CONDITION: [icc] (?!) [x] = [rc](...)
  - BASE-CASE: *proved-under-hypothesis*
    - HYPOTHESIS: [bcc]
    - THESIS: [s]
    - ASSERTION: [bcc] ⇒ [s]
    - PROOF: *list(proof-step ∨ assumption ∨ definition ∨ goal)*
  - INDUCTION-STEP: *proved-under-hypothesis*
    - HYPOTHESIS: [icond] : ([icc] ∧ [ih])
    - THESIS: [s]
    - ASSERTION: [icond] ⇒ [s]
    - PROOF: *list(proof-step ∨ assumption ∨ definition ∨ goal)*

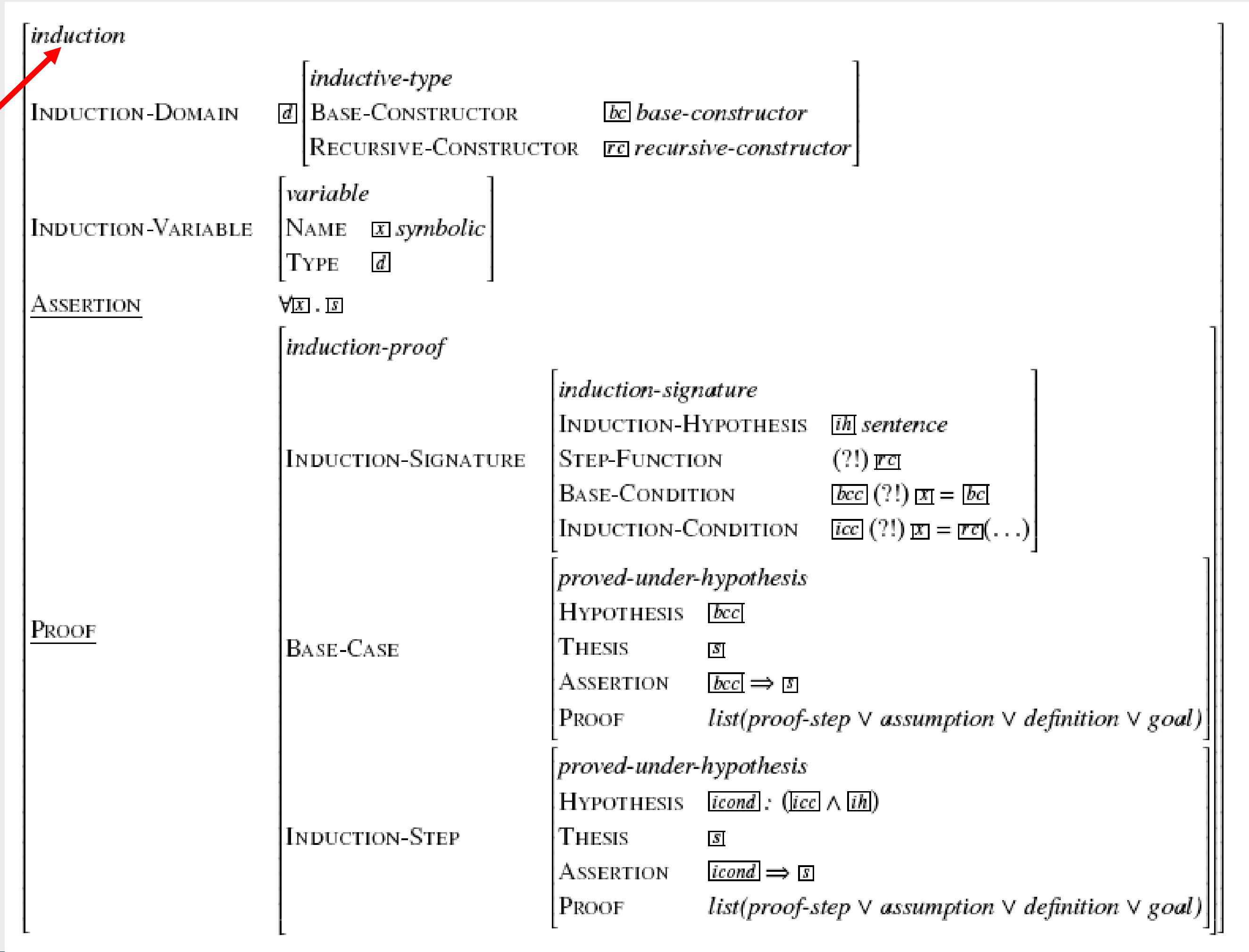
Proof. First, the second statement is indeed more precise than the first: let $k \geq 1$ be such that $f^k = 0$ but $f^{k-1} \neq 0$; there exists $v \neq 0$ such that $f^{k-1}(v) \neq 0$, and we obtain $k \leq n$ by applying the second result to this vector v. We now prove the second claim. Assume therefore that $v \neq 0$ and that $f^k(v) = 0$ but $f^{k-1}(v) \neq 0$. Let $t_0, \ldots, t_{k-1}$ be elements of $\mathbf{K}$ such that

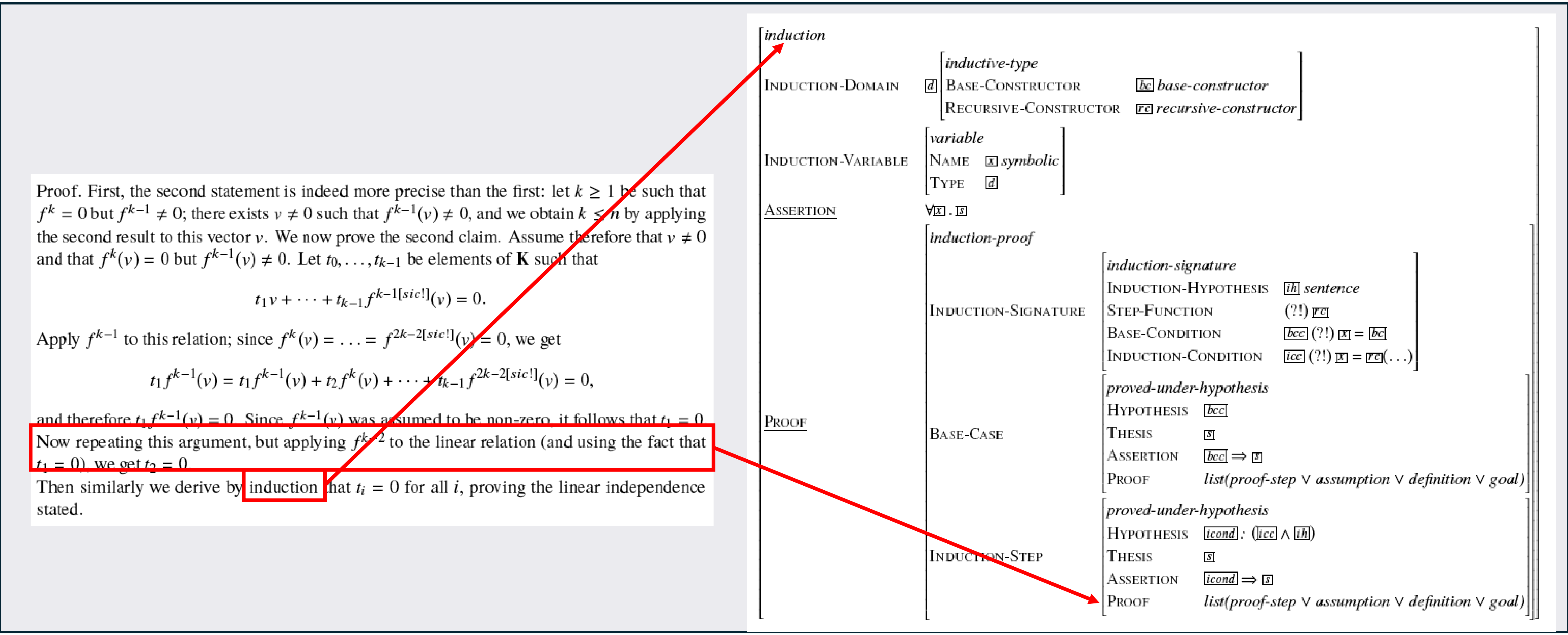
$$t_1 v + \cdots + t_{k-1} f^{k-1[sic!]}(v) = 0.$$

Apply f^{k-1} to this relation; since $f^k(v) = \ldots = f^{2k-2[sic!]}(v) = 0$, we get

$$t_1 f^{k-1}(v) = t_1 f^{k-1}(v) + t_2 f^k(v) + \cdots + t_{k-1} f^{2k-2[sic!]}(v) = 0,$$

and therefore $t_1 f^{k-1}(v) = 0$. Since $f^{k-1}(v)$ was assumed to be non-zero, it follows that $t_1 = 0$. Now repeating this argument, but applying f^{k-2} to the linear relation (and using the fact that $t_1 = 0$), we get $t_2 = 0$.
Then similarly we derive by induction that $t_i = 0$ for all i, proving the linear independence stated.

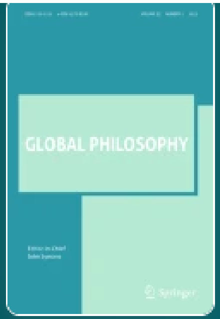



Proof. First, the second statement is indeed more precise than the first: let $k \geq 1$ be such that $f^k = 0$ but $f^{k-1} \neq 0$; there exists $v \neq 0$ such that $f^{k-1}(v) \neq 0$, and we obtain $k \leq n$ by applying the second result to this vector v. We now prove the second claim. Assume therefore that $v \neq 0$ and that $f^k(v) = 0$ but $f^{k-1}(v) \neq 0$. Let $t_0, \ldots, t_{k-1}$ be elements of $\mathbf{K}$ such that
$$t_1 v + \cdots + t_{k-1} f^{k-1[sic!]}(v) = 0.$$
Apply f^{k-1} to this relation; since $f^k(v) = \ldots = f^{2k-2[sic!]}(v) = 0$, we get
$$t_1 f^{k-1}(v) = t_1 f^{k-1}(v) + t_2 f^k(v) + \cdots + t_{k-1} f^{2k-2[sic!]}(v) = 0,$$
and therefore $t_1 f^{k-1}(v) = 0$. Since $f^{k-1}(v)$ was assumed to be non-zero, it follows that $t_1 = 0$.
Now repeating this argument, but applying f^{k-2} to the linear relation (and using the fact that $t_1 = 0$), we get $t_2 = 0$.
Then similarly we derive by induction that $t_i = 0$ for all i, proving the linear independence stated.
induction
INDUCTION-DOMAIN d inductive-type
BASE-CONSTRUCTOR bc base-constructor
RECURSIVE-CONSTRUCTOR rc recursive-constructor
INDUCTION-VARIABLE variable
NAME x symbolic
TYPE d
ASSERTION ∀x . s
PROOF induction-proof
INDUCTION-SIGNATURE induction-signature
INDUCTION-HYPOTHESIS ih sentence
STEP-FUNCTION (?!) rc
BASE-CONDITION bcc (?!) x = bc
INDUCTION-CONDITION icc (?!) x = rc(...)
BASE-CASE proved-under-hypothesis
HYPOTHESIS bcc
THESIS s
ASSERTION bcc ⇒ s
PROOF list(proof-step ∨ assumption ∨ definition ∨ goal)
INDUCTION-STEP proved-under-hypothesis
HYPOTHESIS icond : (icc ∧ ih)
THESIS s
ASSERTION icond ⇒ s
PROOF list(proof-step ∨ assumption ∨ definition ∨ goal)

How to Frame Understanding in Mathematics: A Case Study Using Extremal Proofs

Original Paper | Open access | Published: 05 May 2021

Volume 31, pages 649–676, (2021) Cite this article



Axiomathes

Merlin Carl, Marcos Cramer, Bernhard Fisseni, Deniz Sarikaya & Bernhard Schröder

Abstract

The frame concept from linguistics, cognitive science and artificial intelligence is a theoretical tool to model how explicitly given information is combined with expectations

Aspects of Understanding (1/2)

Understanding a Proof as Constructing an Object

- **Proof Concept O:** object representing a logical deduction of the theorem
- **Understanding O:** constructing the gapless formal object

Aspects of Understanding (2/2)

- **Understanding a Proof as Text Processing**
- **Proof Concept T:** text outlining a proof structure or idea (T)
- **Understanding T:** has components, notably:
  - **TE:** interpreting referring ***expressions***: mathematical areas / objects and relations,
  - **TJ:** understanding the ***justification*** of the proof steps presented in T,
  - **TB:** the ***bridging*** of deductive gaps in T,
  - **TR:** ***recognition*** of the proof method,
  - **TC:** understanding the ***choice***(***s***) of the way of proving in T among possible alternatives

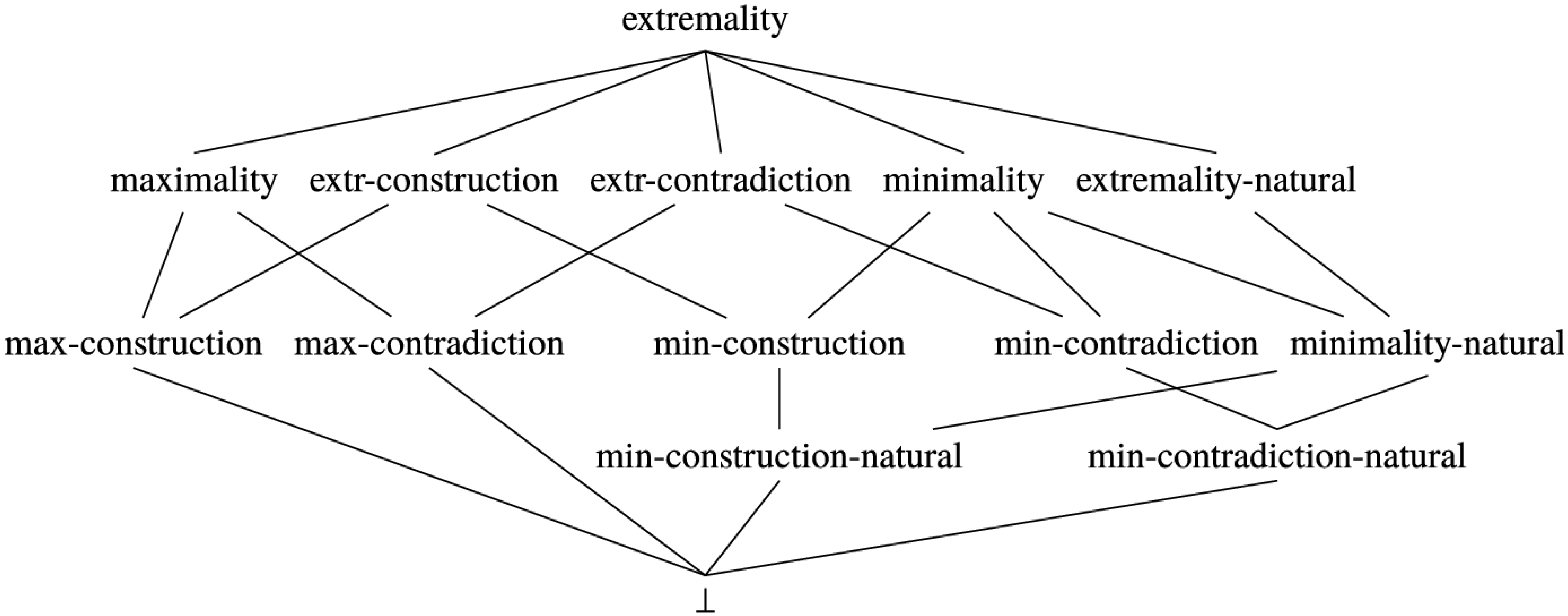
Context and extremal proofs
– preliminary slots

- **Scale:** How are we measuring it?
- **Kind of extremality:** Is it minimal or maximal?
- **Principle evoked for existential claim about extremal object:**
  - least upper bound
  - least number principle
  - ...

Context and extremal proofs – interaction / ontological frames

- *Das Extremalprinzip setzt also einen Kontext voraus, in dem minimale oder maximale Objekte existieren.“* Carl 2017

- **Variations of extremal proofs**
- **Carl**: variation triggered by (**Engel**’s “three well-known facts”), e.g.,
  domain natural numbers: triggers *least number principle*
  domain subset of reals: triggers *least upper bound principle* or
  largest lower bound principle



extremality
maximality
extr-construction
extr-contradiction
minimality
extremality-natural
max-construction
max-contradiction
min-construction
min-contradiction
minimality-natural
min-construction-natural
min-contradiction-natural
⊥

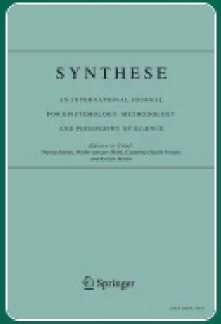
Example 3: Research Level

- The aim of this thesis is to present new method based on algebraic and analytic tools – the celebrated method of flag algebras invented by Razborov [67]. This method provides a uniform framework for standard counting techniques used in extremal combinatorics. It is inspired by the theory of dense graph limits, on which we focus in Chapter 4. Despite the fact that the method is quite new, it has been successfully applied to various problems in extremal combinatorics, giving solutions to many long open-standing problems. In particular in Turán-type problems in graphs [23, 35, 39, 41, 61, 63, 64, 70, 74, 76], 3-graphs [7, 27, 28, 32, 62, 69], and hypercubes [5, 8], Caccetta-Häggkvist conjecture [42, 71], extremal problems in a colored setting [6, 22, 38, 50], and in geometry [51]. More details on these applications can be found in a recent survey of Razborov [68]. (Grzesik 2014, p. 2)

How to frame innovation in mathematics

Original Research | Open access | Published: 25 September 2023

Volume 202, article number 108, (2023) Cite this article



Bernhard Fisseni, Deniz Sarikaya & Bernhard Schröder

Part of a collection:

Linguistically informed Philosophy of Mathematics: How the study of mathematical texts contributes to the investigation of philosophical problems

Abstract

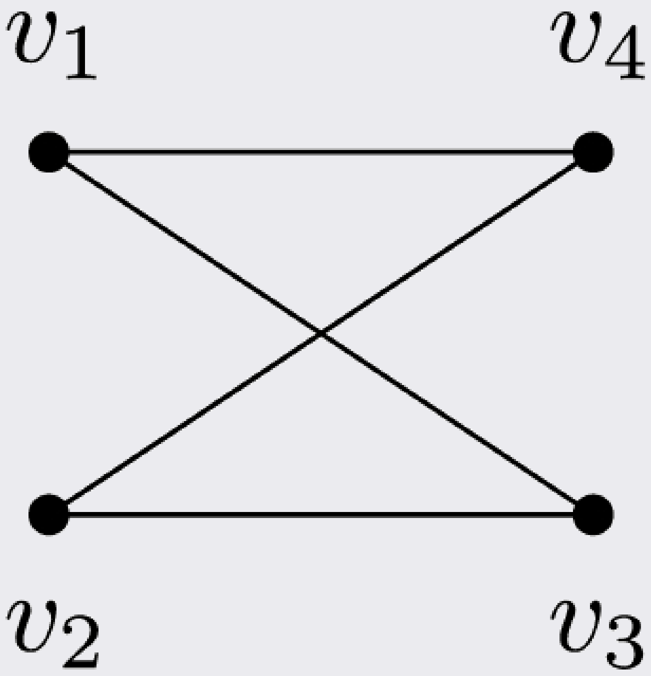
We discuss conceptual change and progress within mathematics, in particular how tools, structural concepts and representations are transferred between fields that appear to be

Bridges in Mathematics

- Hillel Furstenberg and Gregory Margulis – “for pioneering the use of methods from probability and dynamics in group theory, number theory and combinatorics”.
- Akshay Venkatesh – “for his synthesis of analytic number theory, homogeneous dynamics, topology, and representation theory, which has resolved long-standing problems in areas such as the equidistribution of arithmetic objects.”
- *Langlands Program*, Lafforgue in 2002 or Ngô in 2010.

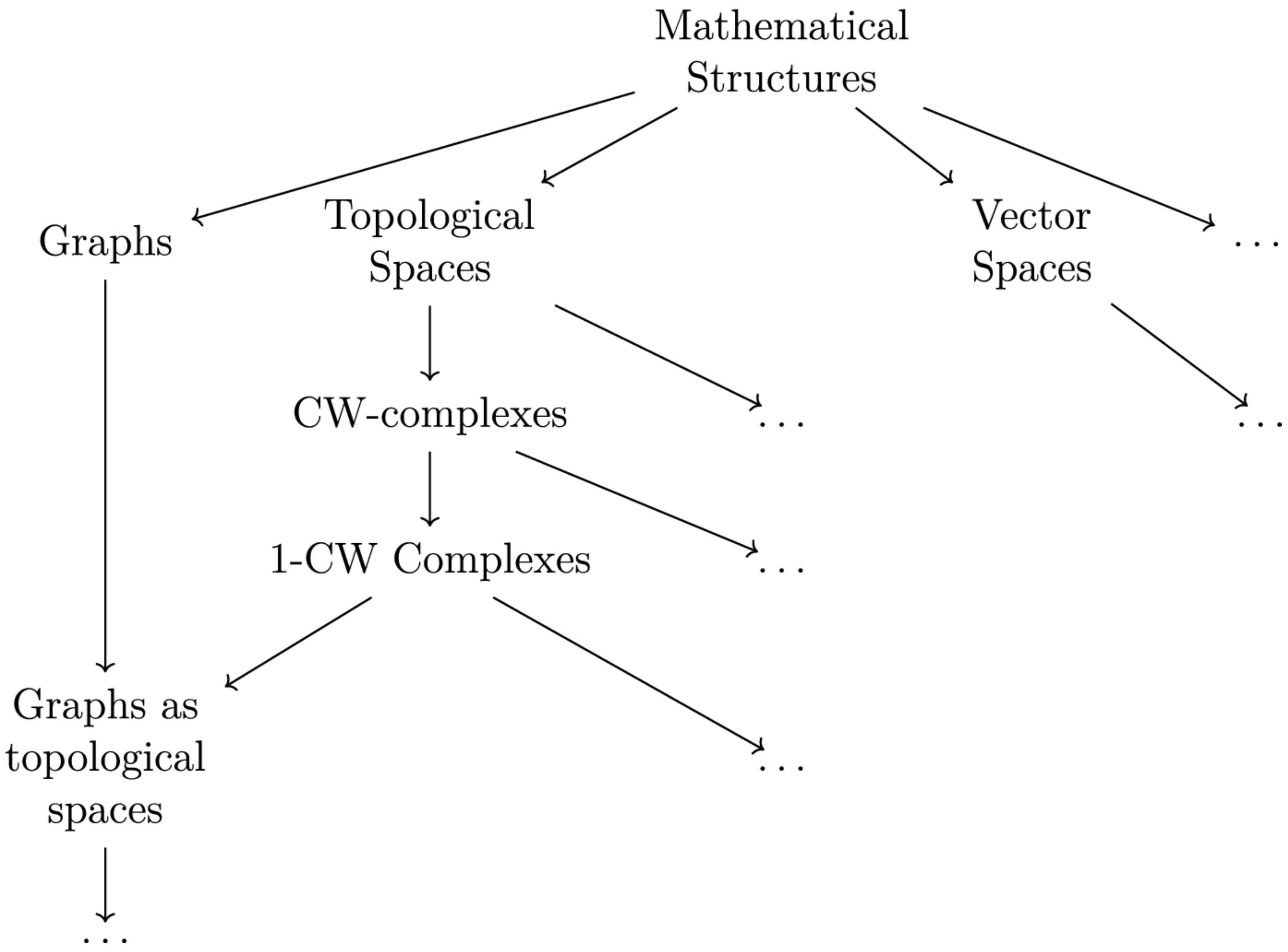
A graph: Algebraic vs. Combinatorical

$$a = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

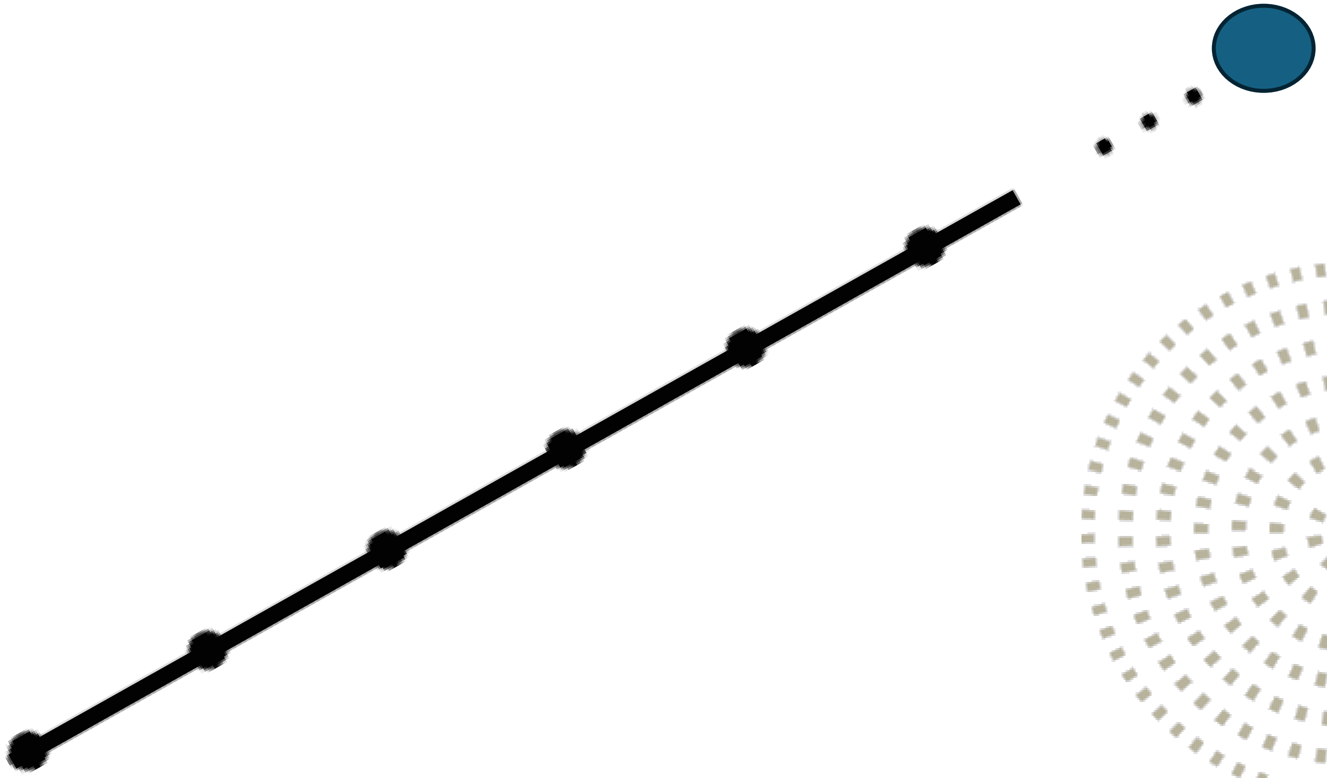


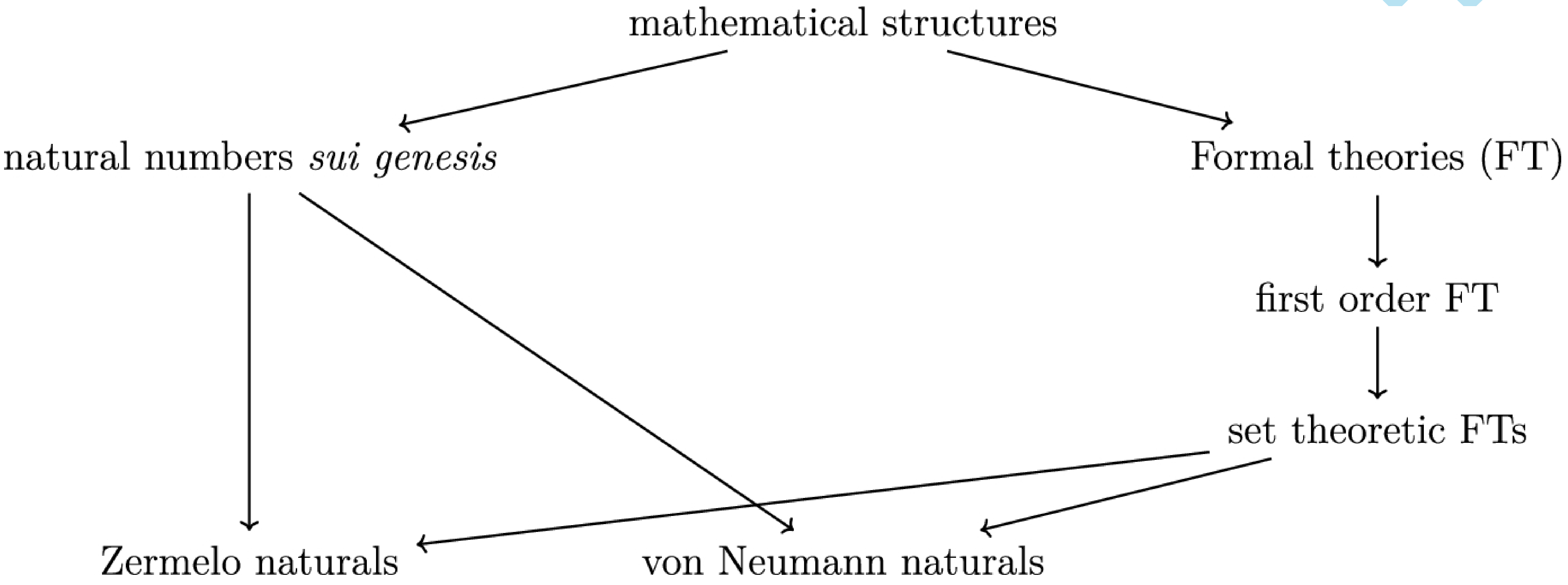
Raises different Questions: e.g. Eigenvalues

A part of the frame hierarchy of mathematical objects, containing both topological spaces and graphs



Graphs as topological objects




mathematical structures
natural numbers *sui genesis*
Formal theories (FT)
first order FT
set theoretic FTs
Zermelo naturals
von Neumann naturals

Future Work

- Linguistic: Annotation Workflow => More Frames
- Math Education: Are frames usefull in teaching?
- Philosophical
  - Question of style?
  - Understanding in the hermeneutic tradition, why did the author wrote this? Embedding in socio-historical context
- Computer Science: Implementation in theorem proving software
- …

THANK YOU

Committee: Karen François, Joachim Frans, **Ulrich Gähde**, Valeria Giardino, Jean Paul Van Bendegem, and **Bart Van Kerkhove**

Family: Mira, Ali, Sükran, Laila, and Mian; honorary thanks to Nara

Coauthors: Merlin Carl, Marcos Cramer, Bernhard Fisseni, Karl Heuer, Deborah Kant, José A. Pérez-Escobar, and Bernhard Schröder

Feedback: Ozan A. Altinok, Neil Barton, Balthasar Grabmayr, David A. King, Lukas Lang, Alexander Okupnik, Hannah Pillin, Sophie Nagler, Armin Rezaei Nia, Colin J. Rittberg, Mira Sarikaya, Roy Wagner, and Clemens W. Wetcholowsky

Hosts: Benedikt Löwe, Pascal Gollin, Colin J. Rittberg, and Roy Wager

Funding and ideal support: Claussen-Simon-Stiftung, Studienstiftung des deutschen Volkes, FWO (for offsprings of the thesis) .

QUESTIONS / COMMENTS